

國立屏東教育大學 102 學年度研究所碩士班入學考試

微積分 (B) 試題

(應用數學系碩士班)

※請注意：答案須寫在答案卷上，否則不予計分。

計算題 (每題 10 分，共 100 分)

1. Find $\frac{dy}{dx}$ given that $x \sin y - y \sin x = 0$
2. Find $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$.
3. Let $f(x) = \frac{c}{x} + x^2$. Determine all values of the constant c such that f has a relative minimum, but no relative maximum.
4. Find $\int \frac{2x}{(x+1)^2} dx$.
5. Find $\int \frac{dx}{\sqrt{e^{2x} - 1}}$.
6. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$.
7. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$.
8. Suppose that the interval of convergence of the series $\sum_{n=0}^{\infty} a_n x^n$ is $(-4, 4)$. What is the radius of convergence for $\sum_{n=0}^{\infty} a_n x^{2n}$.
9. Find the arc length of the graph of $y = \frac{1}{2}x^2$ from $x = 0$ to $x = 1$.
10. Evaluate the integral $\iint_R (x+y)^2 \sin^2(x-y) dA$, where R is the region bounded by the square with vertices at $(\pi, 0)$, $(3\pi/2, \pi/2)$, (π, π) and $(\pi/2, \pi/2)$.