

國立屏東教育大學 99 學年度研究所碩士班入學考試

微積分(A) 試題

(應用數學系碩士班)

※請注意：答案須寫在答案卷上，否則不予計分。

計算題 (每題 10 分，共 100 分)

1. $\lim_{x \rightarrow \infty} \frac{3x-2}{\sqrt{2x^2+1}}$

2. Solve $y' = \frac{2x}{y}$

3. $\int 5xe^{-x^2} dx$

4. Find arclength of $y = \frac{x^3}{6} + \frac{1}{2x}$ on $[\frac{1}{2}, 2]$.

5. Find $\sum_{n=1}^{\infty} \frac{2}{4n^2-1}$

6. Find the extreme of $f(x) = 2\sin x - \cos 2x$ on the interval $[0, 2\pi]$.

7. Find the values of p for which the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges.

8. Let $w = f(x, y)$, where $x = r \cos \theta$ and $y = r \sin \theta$, show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta}\right)^2.$$

9. Find the radius of convergence and interval of convergence of the infinite series $\sum_{n=1}^{\infty} \frac{(-2)^n x^n}{\sqrt[4]{n}}$.

10. Find the values of a and b such that $\lim_{x \rightarrow 0} \frac{a - \cos bx}{x^2} = 1$.