

國立屏東教育大學 103 學年度學士班轉學考試

線性代數 試題

(應用數學系)

*注意事項：

- (1) 本試題共 2 頁，答案請「橫式」書寫，並依規定上下翻頁。
(2) 不必抄題，但請依序將題號標出，並寫在答案紙上，否則不予計分。

一、計算題 (共 100 分)

1. Let $T: V \rightarrow W$ be a linear transformation and $\dim V = \dim W$. Show that T is one to one if and only if T is onto. (15%)
2. Let A, B be $m \times n$ matrices. Show that $|r(A) - r(B)| \leq r(A + B) \leq r(A) + r(B)$, where $r(A), r(B)$ are ranks of A, B . (10%)
3. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$. Show that A and B are not similar and they have different characteristic polynomials but have the same minimum polynomial. (15%)
4. Let $A = \begin{pmatrix} 1 & 0 \\ 2 & \frac{1}{2} \end{pmatrix}$. Find $e^A = ?$ (10%)
5. Find the volume of the parallelepiped with a vertex at the origin and edges $u = i - 2j + 3k$, $v = i + 3j + k$, $w = 2i + j + 2k$. (10%)
6. Determine if the following polynomials are linear independent in P_2 . If not, express one vector as a linear combination of the rest. $\{t^2 - 4, 5t^2 - 5t - 6, 3t^2 - 5t + 2\}$ (10%)
7. Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of A . (10%)

8. Let

$$A = \begin{bmatrix} 6 & 1 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 5 & 9 & -3 & 1 \\ 4 & 1 & -3 & 2 \end{bmatrix}.$$

Compute the determinant of A.

(10%)

9. Let

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}.$$

Solve the 2×2 system of differential equations: $\mathbf{x}' = A\mathbf{x}$.

(10%)